C2M10

Sequences

When your instructor told you that a sequence is a function whose domain is the natural numbers, or a subset thereof, it is possible that you did not attach as much importance to that idea as you did to the mechanics of dealing with sequences. Maple allows us to define expressions and functions, and it is sometimes confusing as to which we want to use. We will define the sequence $\{a_n\} = \left\{\dfrac{n+2}{3n-1}\right\}$ as an expression and the sequence $\{b_n\} = \left\{\sqrt{n^2+3n}-n\right\}$ as a function to illustrate how they must be handled differently.

Maple Example 1: $\{a_n\} = \left\{\dfrac{n+2}{3n-1}\right\}$

```
> with(student):
> a:=(n+2)/(3*n-1);
```

$$a := \frac{n+2}{3n-1}$$

```
> seq(a,n=1..10);
```

$$\frac{3}{2}, \frac{4}{5}, \frac{5}{8}, \frac{6}{11}, \frac{1}{2}, \frac{8}{17}, \frac{9}{20}, \frac{10}{23}, \frac{11}{26}, \frac{12}{29}$$

```
> limit(a,n=infinity);
```

$$\frac{1}{3}$$

Maple Example 2: $\{b_n\} = \left\{\sqrt{n^2+3n}-n\right\}$

```
> with(student):
> b:=n->sqrt(n^2+3n)-n;
```

$$b := n \to \sqrt{n^2+3n}-n$$

```
> seq(b(n),n=1..10);
```

$$1, \sqrt{10}-2, 3\sqrt{2}-3, 2\sqrt{7}-4, 2\sqrt{10}-5, 3\sqrt{6}-6, \sqrt{70}-7, s\sqrt{22}-8, 6\sqrt{3}-9, \sqrt{130}-10$$

```
> limit(b(n),n=infinity);
```

$$\frac{3}{2}$$

To understand this last limit, consider multiplying b_n by its conjugate, and then dividing by it.

$$\left(\sqrt{n^2+3n}-n\right)\cdot\frac{\sqrt{n^2+3n}+n}{\sqrt{n^2+3n}+n} = \frac{n^2+3n-n^2}{\sqrt{n^2+3n}+n} = \frac{3n}{\sqrt{n^2+3n}+n}\cdot\frac{1/n}{1/n} = \frac{3}{\sqrt{1+3/n}+1} \to \frac{3}{2}$$

with the limit taken as $n \to \infty$.

The sequence $\{a_n\}$ is obtained from an expression whose name is a while the sequence $\{b_n\}$ is obtained by evaluating a function whose name is b. If we had the command `seq(b,n=1..10);` what would we have obtained? The answer - ten b's, because the function b must be evaluated in order for it to have a value. This will be very important in the next section when we will need to consider the term $\dfrac{b_{n+1}}{b_n} = \dfrac{b(n+1)}{b(n)}$. It would be cumbersome and less clear to find $\dfrac{a_{n+1}}{a_n}$ when a is an expression. The command would be `subs(n=n+1,a)/a;`.

C2M10 Problems Using Maple, find the first ten terms of each sequence and the limit of each.

1. $a_n = \left\{\dfrac{n^2}{3^n}\right\}$
2. $b_n = \left\{\dfrac{n^2-3n+4}{5+2n+6n^2}\right\}$
3. $c_n = \left\{\left(1+\dfrac{2}{n}\right)^n\right\}$
4. $d_n = \left\{\left(1-\dfrac{2}{n}\right)^n\right\}$
5. $e_n = \left\{\sqrt{n^2+6n}-n\right\}$